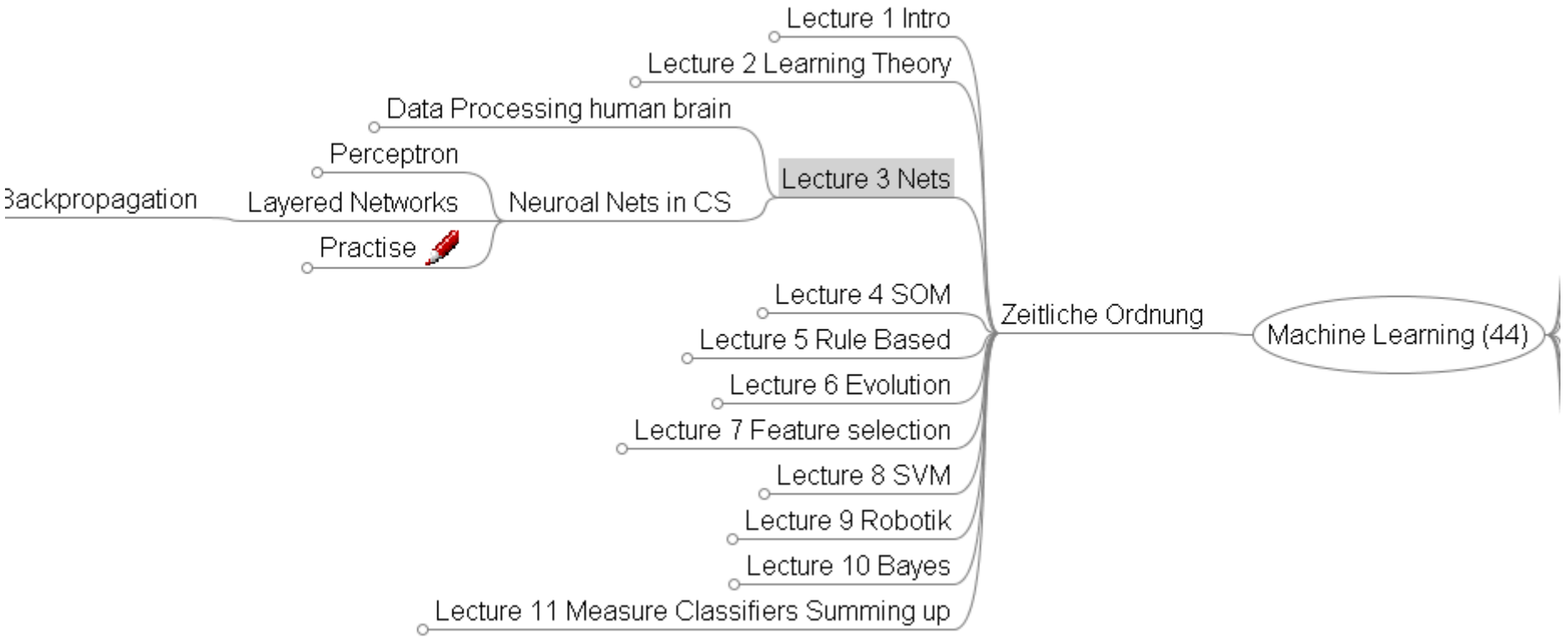
Tree-like structures

break down high complexity into smaller parts

Drill-Down Search Menu

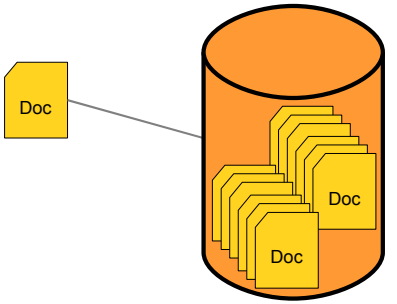
- Structures complexity of product variants

Lecture planning

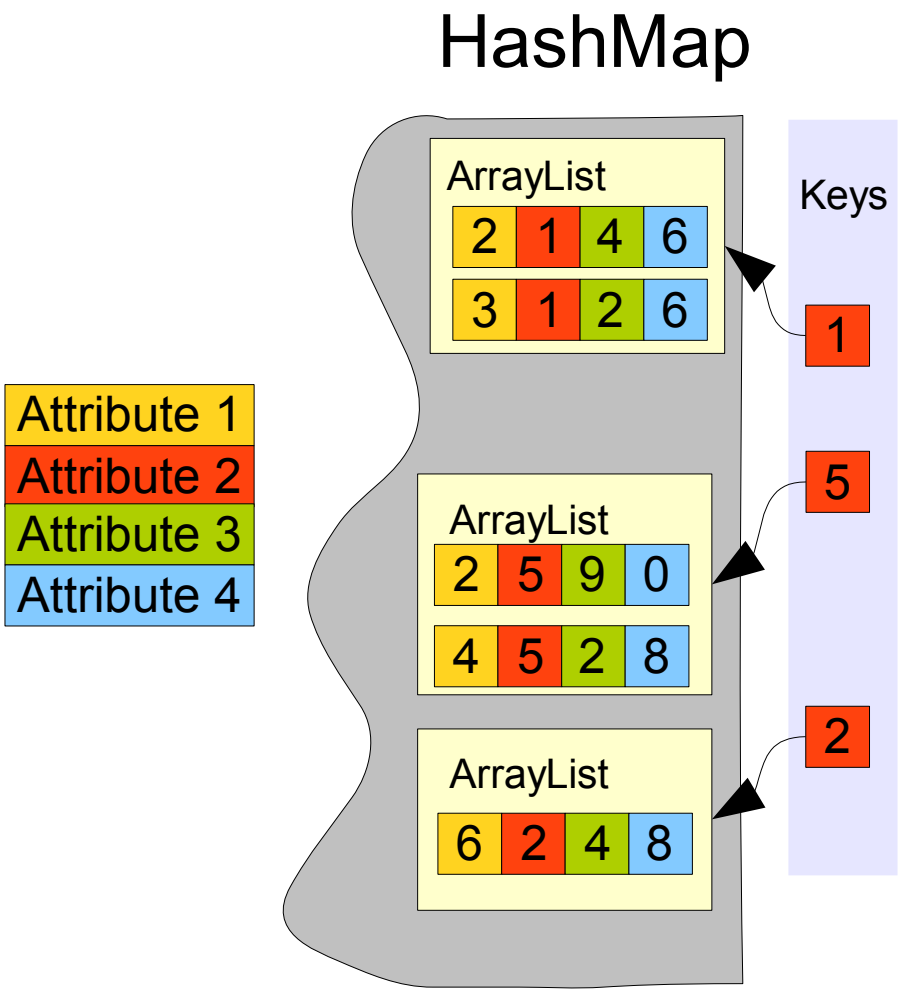


Index into Databases

- Maintain low access times even if the database is “loaded”

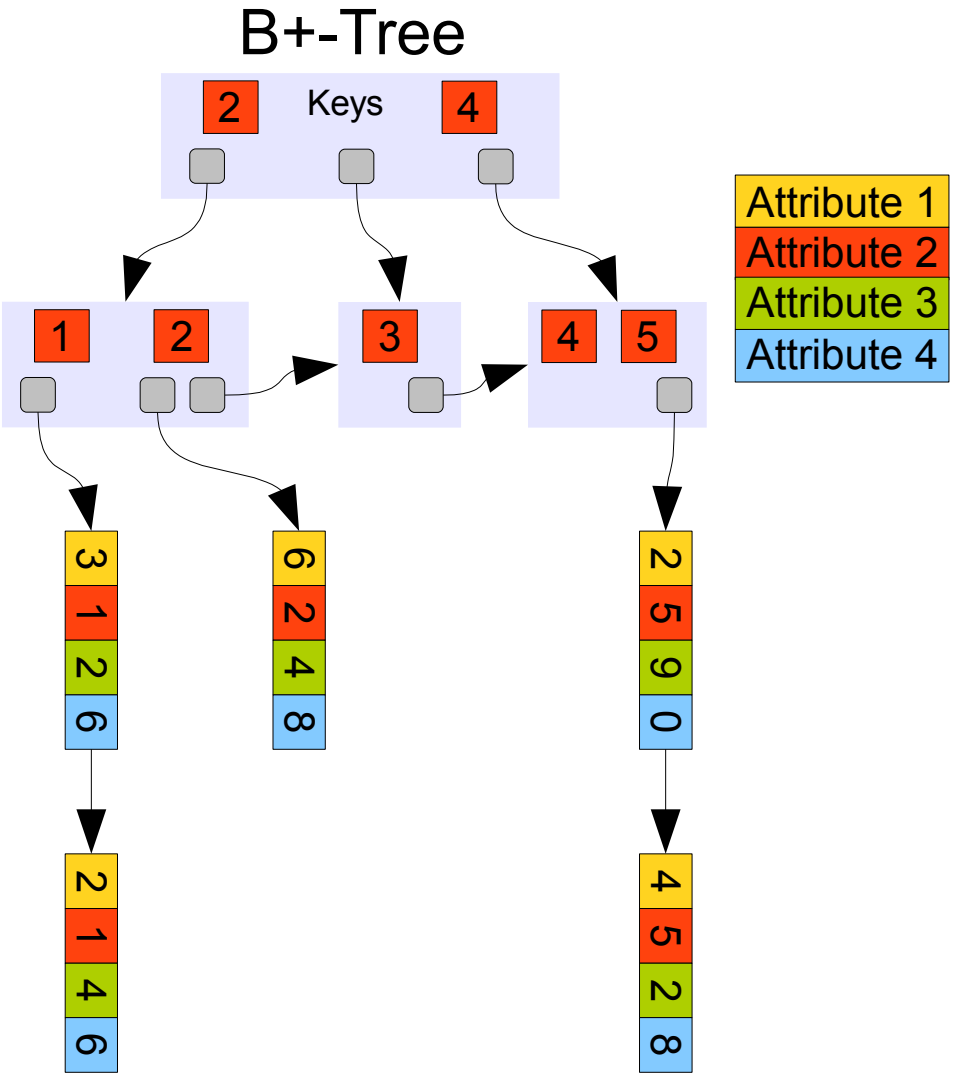


- Hash-Maps
  - One-step access
  - Is **not** applicable for comparable conditions (>,<)



Support for comparison

- B+tree
  - Keys are sorted
- Samples are sorted at time of insertion
- For char/text attributes use a index length



B+Tree

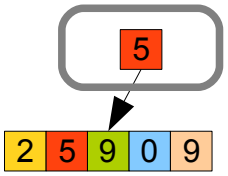
- Contains buckets
  - The tree consists of
    - Inner nodes
      - Contain only keys and pointers-to-buckets
    - End nodes (the leaves)
      - Contain keys, pointers-to-buckets and pointers-to-data
  - Each buckets contains
    - between $n/2 \leq m \leq n$ keys
    - m+1 pointers-to-buckets or m pointers-to-data (only leaves)
    - The keys in the bucket are ordered

Creating a B+Tree

- Create a B+Tree for the following data using attribute 2 (red) and n=3
  - The insertion order is from top to bottom

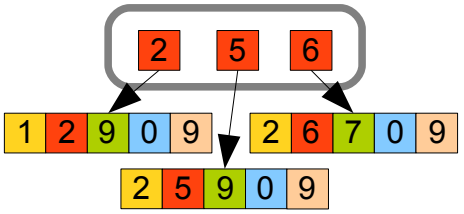
Samples	Keys
2 5 9 0 9	5
2 6 7 0 9	6
1 2 9 0 9	2
1 4 5 0 9	4
4 8 2 0 1	8
4 9 9 0 1	9

1. Insert “5”



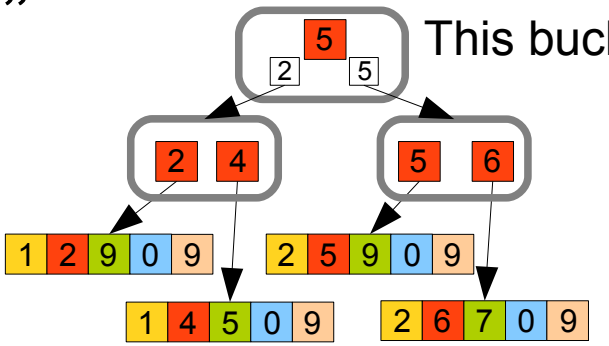
This bucket contains a key and a pointer to a data sample

2. Insert “6”,”2”



Maximal size for the bucket is reached!

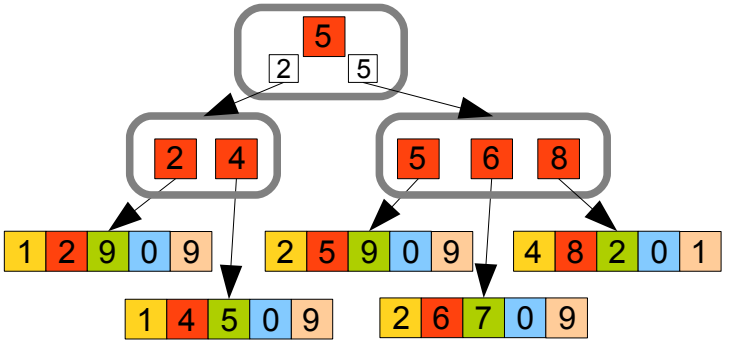
3. Insert “4”



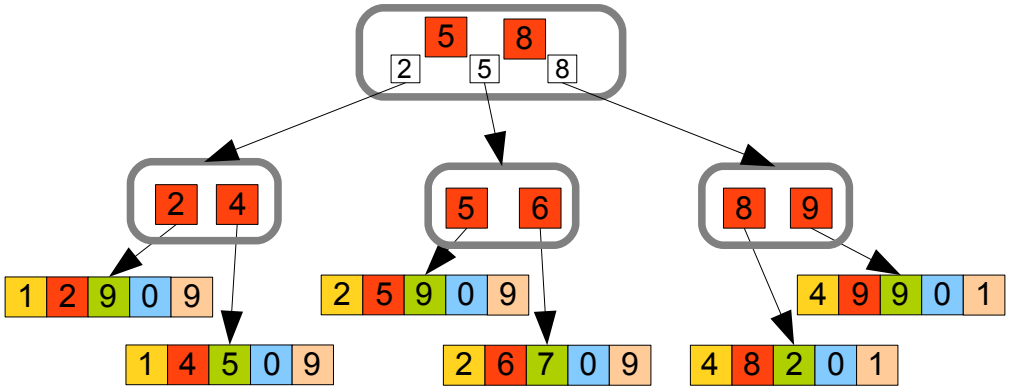
This bucket contains a pointers to other buckets

Creating a B+Tree

4. Insert “8”

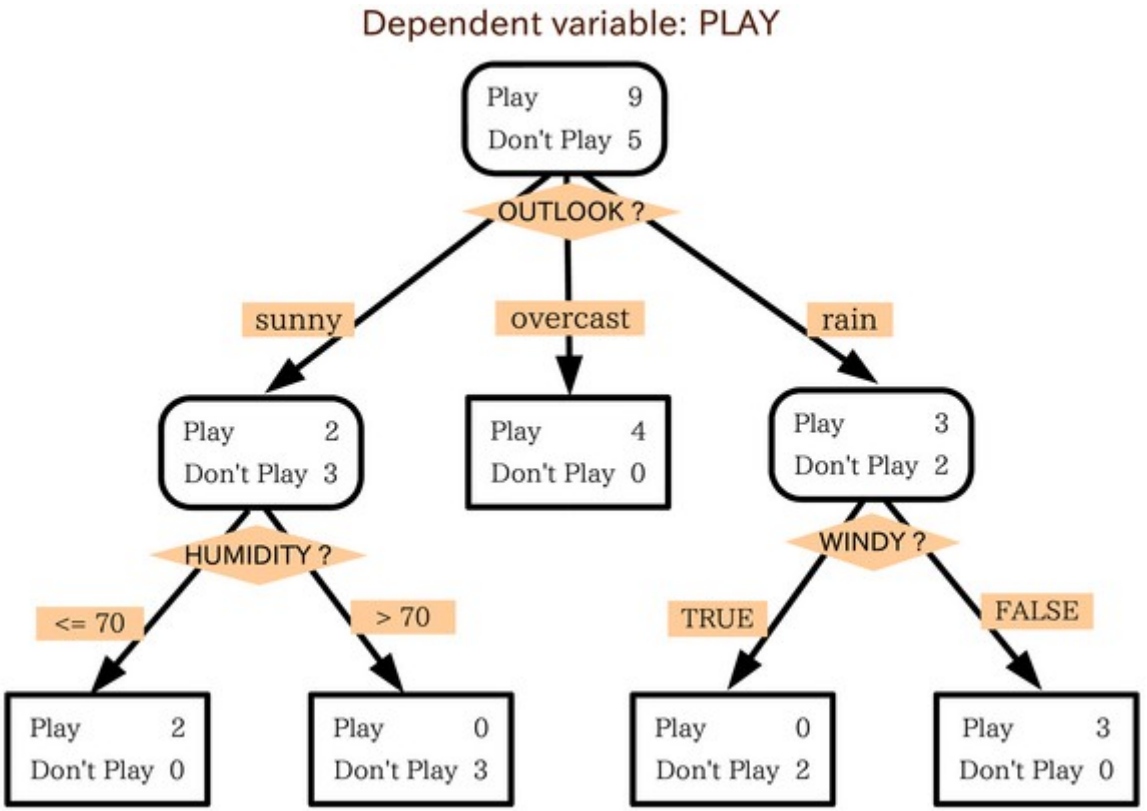


5. Insert “9”



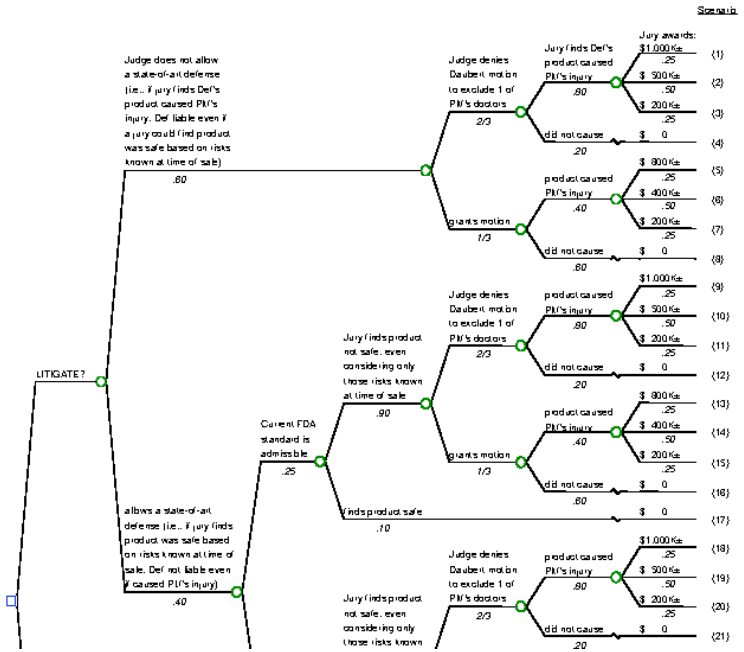
- If a bucket splits
  - the middle key travels “up” to the parent
  - The pointers are named according to the smallest key in the bucket to which the pointer points

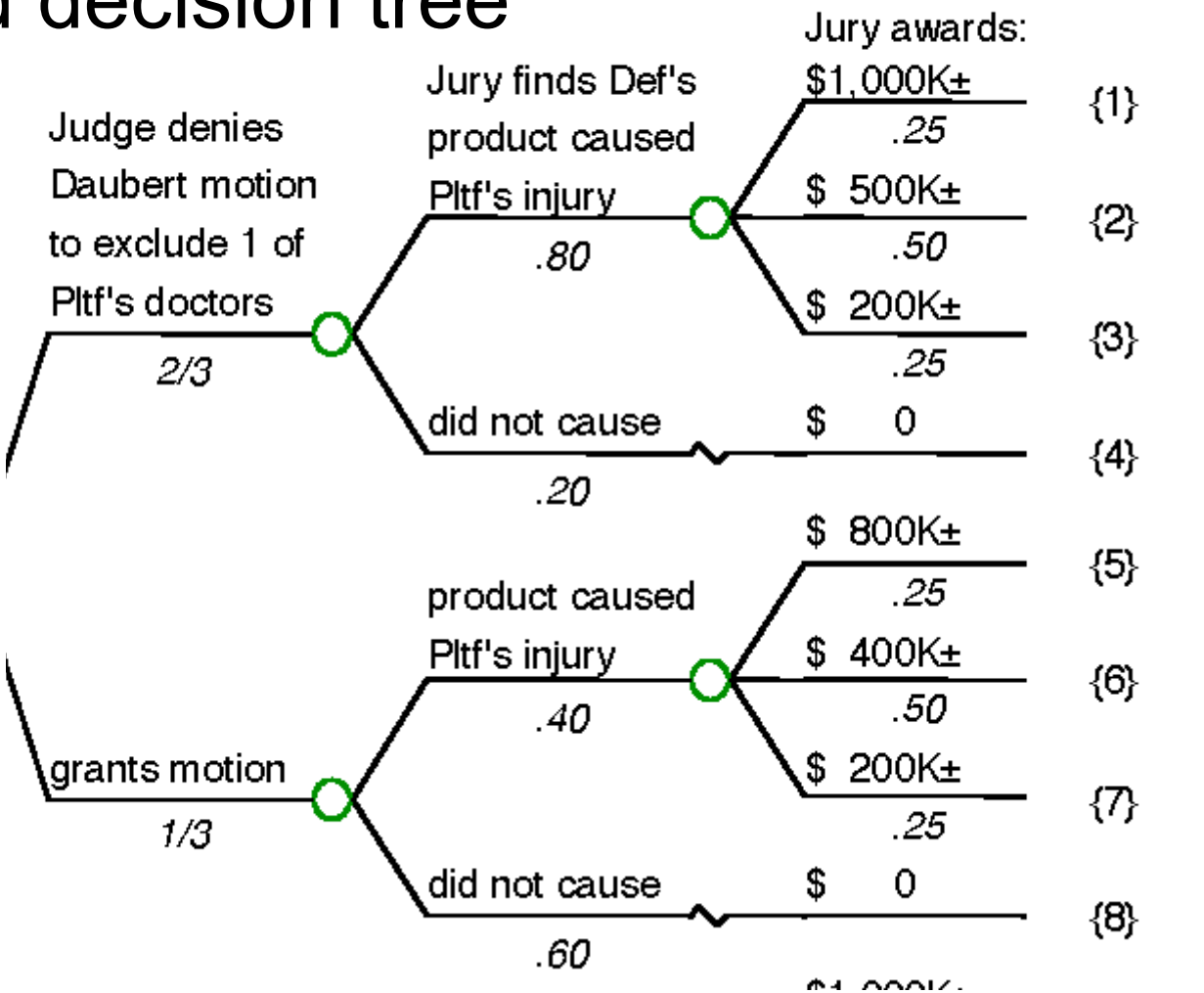
Decision making



Litigation

- Strategical decision making
- A fully developed decision tree



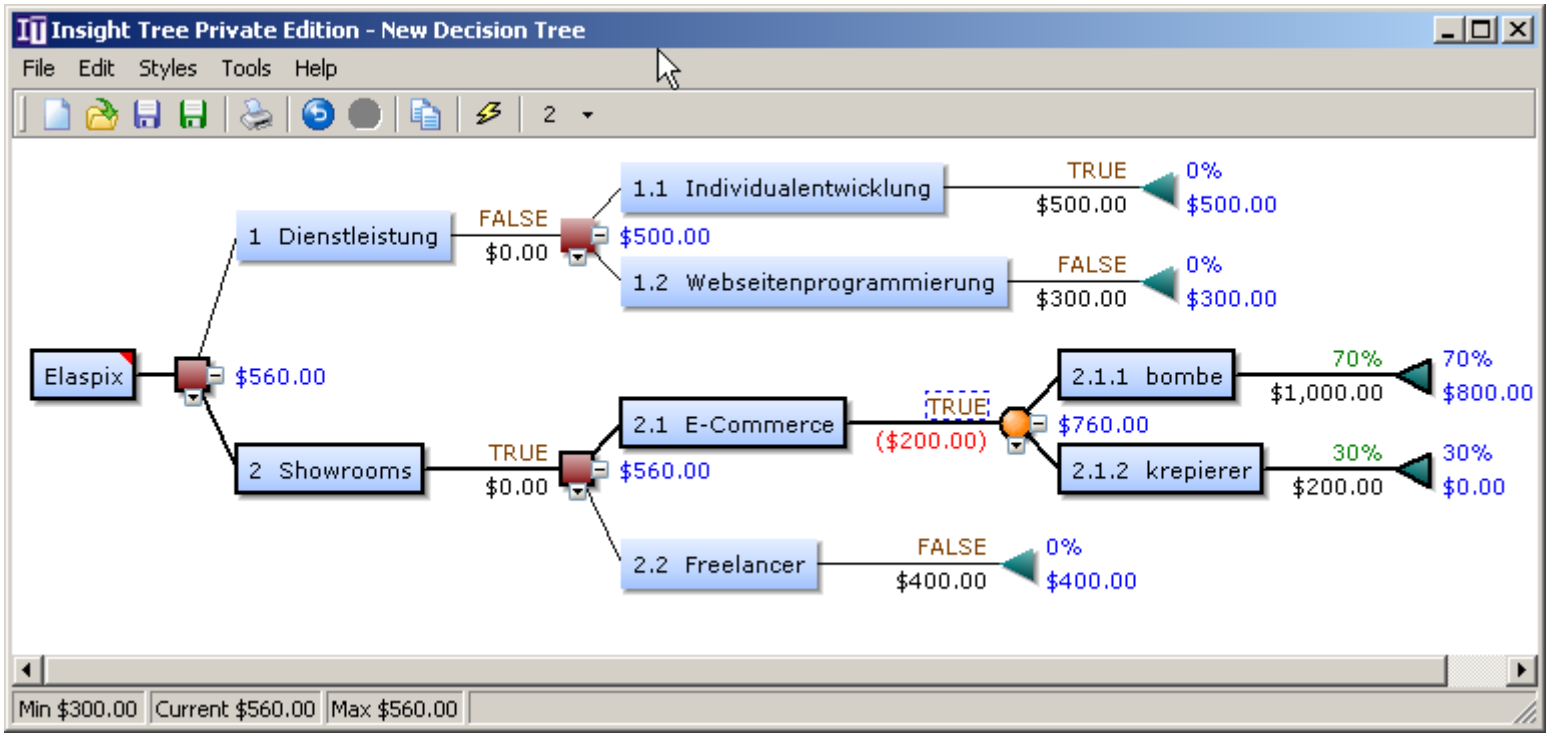


Decision Tree Analysis

- Find answers to a multiple-choice question with several option for each question
- Incrementally develop the tree
  - From Top to Bottom
  - Think local is much easier than thinking global
- Easy to use and understand
- Deal with uncertainty

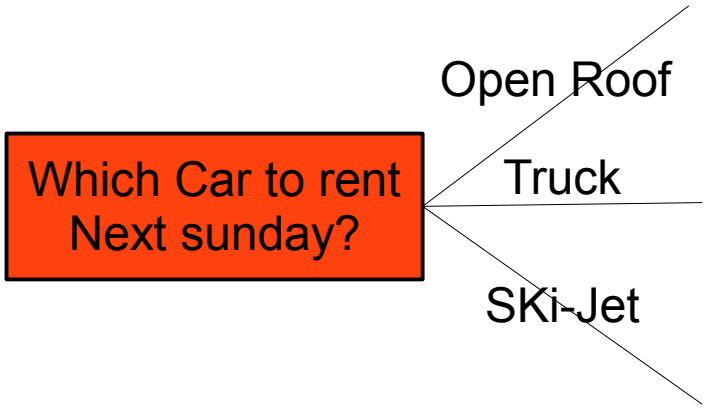
Example Software

- http://www.visionarytools.com/



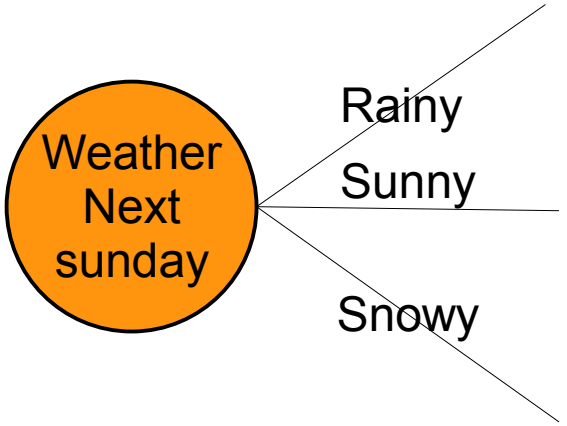
Your own Decisions

- A decision about a topic is drawn as square
- For each alternative create new sub-Trees



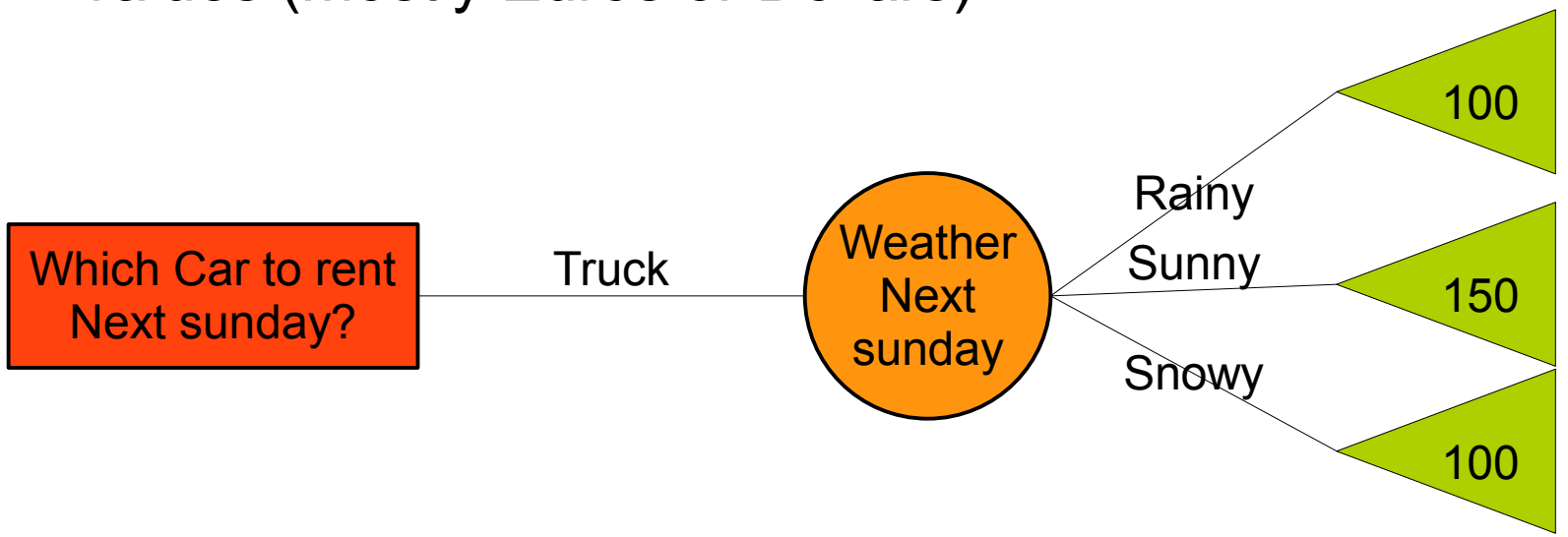
Uncertainties beyond your control

- Other decisions or Uncertainties are drawn as circle
  - How many outcomes may be possible?
    - Each form a new sub-tree



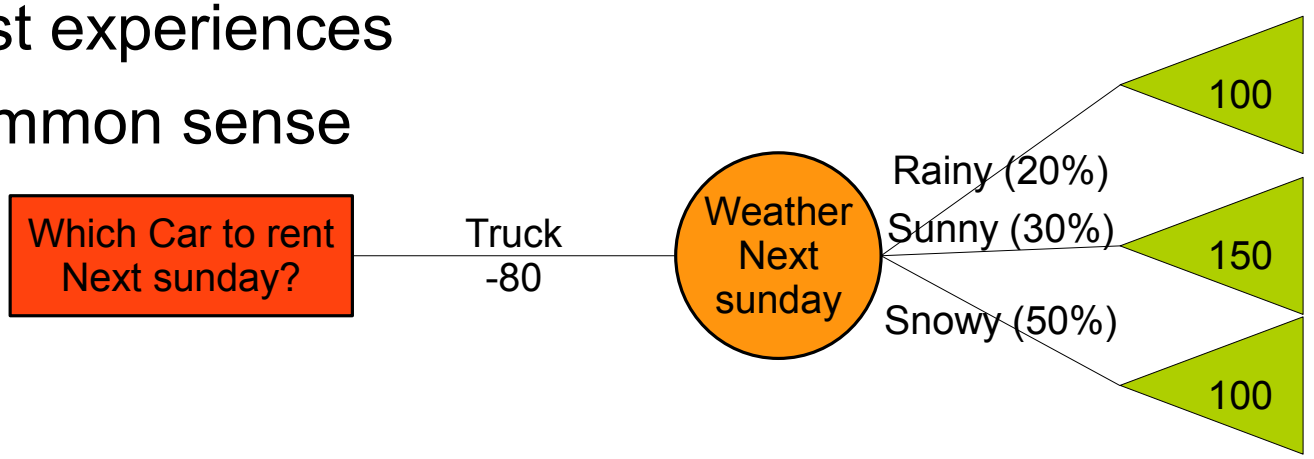
Make sub Trees until you are done

- At each line you have 3 options
  - Decision
  - Uncertainty
  - Outcome
    - may be postive or negative expressed in numerical values (mostly Euros or Dollars)

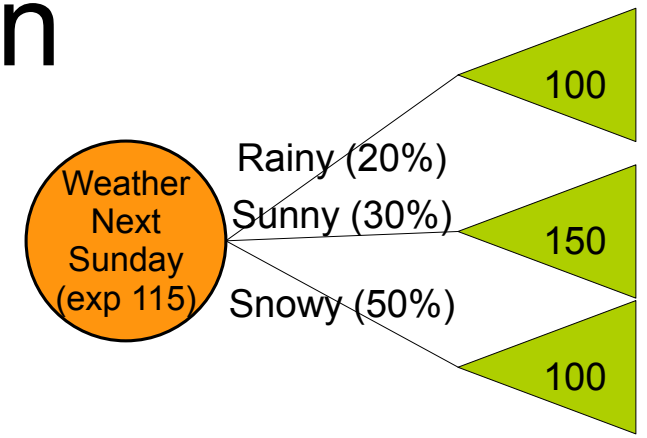


Make it more plastic using numbers

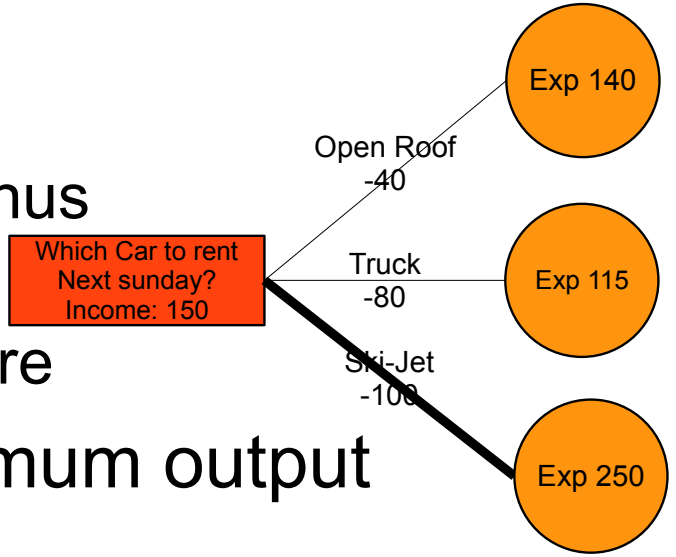
- To the branches of decisions add
  - Guaranteed Costs
- To the branches of uncertainties add
  - Likelihoods (sum up to 100%)
  - Those likelihoods are coming from
    - Market research
    - Past experiences
    - Common sense



Computation



- From right to left
  - For uncertainties
    - Multiply the outcome of each leaf with its probability and sum them up
    - Write the sum into the circle
  - For decisions
    - Compute: multiplication result minus the costs
    - Write the maximum into the square
  - Choose the path with the maximum output



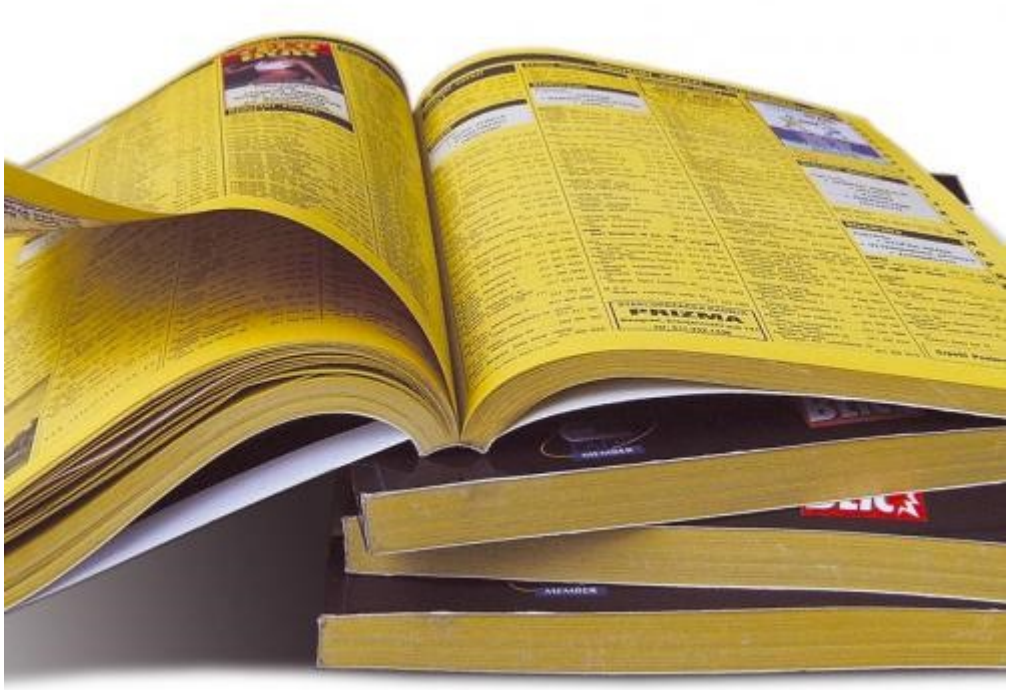
Induction of decision trees

- To understand large amounts of nominal data
  - Build an tree automatically and extract the paths as rules
  - Interprete the rules to understand the data
- To create prediction models from training data
  - Given: supervised samples with class and nominal attributes
  - Task: create a decision tree such that the class of new unseen samples can be predicted from their known attributes

Telephone book

- Stores “observations” in a alpha-numerical order

- What is the worst case number of queries if the telephone book is not ordered?
- What is the worst case number of queries when ordered?



http://www.calgary-city-maps.com/images/Calgary_phone_book.jpg

Concept of Entropy

- A measure of how much confusion (deutsch Unordnung) a system has
  - Entropy of zero means: no confusion
- A measure of how much uncertainty an event has
  - Entropy of zero has no uncertainty

http://www.unserekirche.de/data/images/520/245047771580.jpg

Information Entropy

- Derived from telecommunication engineering
  - How many bits are required to encode a message
  - e.g. The symbols transmitted over a network are only (a_1,a_2,a_3,a_4), (could mean: a_1=start,a_2=stop, a_3=reset, a_4=quit)
  - How many bits are required to encode the following messages? (the transmission time doesn't matter, only the number of different symbols are important, you have an extra signal for free that indicates that a new message is incoming)
    - Message 1: {a_1,a_1,a_1,a_1,a_1,a_1,a_1}
    - Message 2: {a_1,a_1,a_1,a_2,a_2,a_2}
    - Message 3: {a_1,a_2,a_3,a_4}

How many bits are needed

- How many bits are required to encode the following messages? (the transmission time doesn't matter)
  - Message 1: $\{a_1,a_1,a_1,a_1,a_1,a_1,a_1\}$ Bits=0
  - Message 2: $\{a_1,a_1,a_1,a_2,a_2,a_2\}$ Bits=1
  - Message 3: $\{a_1,a_2,a_3,a_4\}$ Bits=2
- Alternative question
  - How many Yes/No Questions are required to encode the message
  - How strong it is to guess / predict the message
    - The more bits the more difficult is the quest

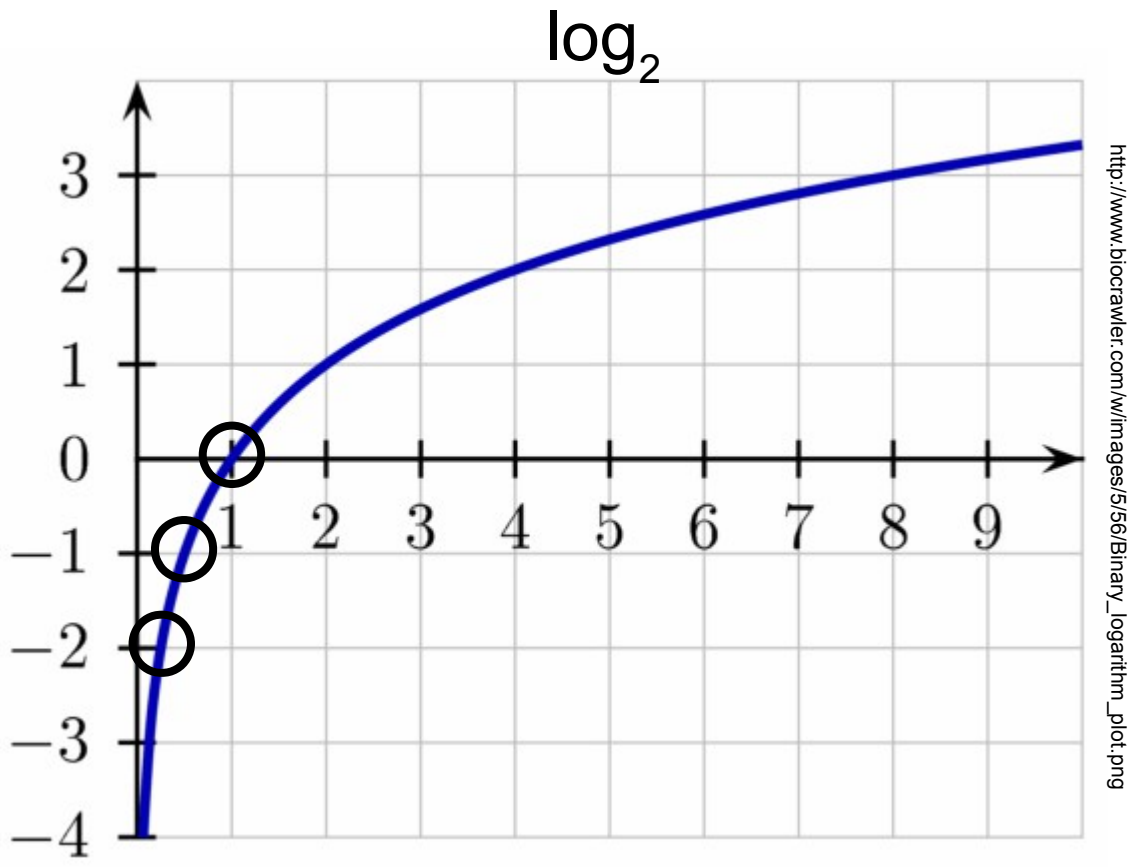
The magic function

- Which function is best appropriate to compute the number of bits dependent on frequency
- Based on frequency freq(a_i) of message values
- Need a function f with properties
  - f(1)=0 and f(0.5)=1 and f(0.25)=2

Message	freq (a_1)	freq (a_2)	freq (a_3)	freq (a_4)	Entropy
$\{a_1,a_1,a_1,a_1,a_1,a_1,a_1\}$	1	0	0	0	0
$\{a_1,a_1,a_1,a_2,a_2,a_2\}$	0.5	0.5	0	0	1
$\{a_1,a_2,a_3,a_4\}$	0.25	0.25	0.25	0.25	2

... using the binary logarithm

- $f=-\log_2(freq)$



http://www.biocrawler.com/w/images/5/56/Binary_logarithm_plot.png

Computing the entropy

- E is the entropy (how many bits are needed)
- M is the message containing the symbols
- n is the number of defined symbols
- p_{ai} is the frequency of the a_i-th symbol

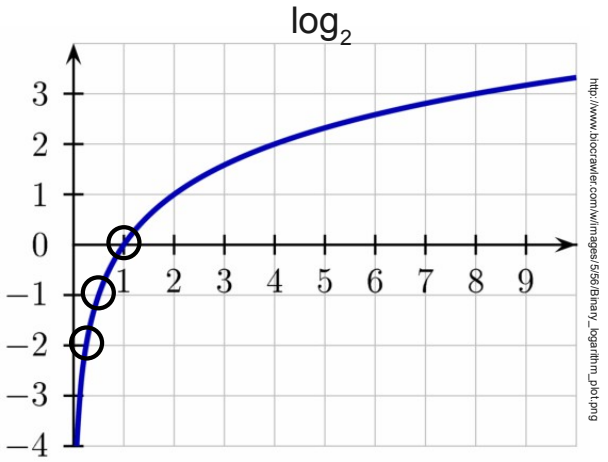
$$E(M) = -\sum_{i=1}^{n} p_{a_i} \cdot \log_2(p_{a_i})$$

e.g. E({a_1,a_1,a_1,a_2,a_2,a_2})=1 Bit

$$E(\{a_1, a_1, a_1, a_2, a_2, a_2\}) = -p_{a_1} \cdot \log_2(p_{a_1}) - p_{a_2} \cdot \log_2(p_{a_2})$$

Example

$$E(M) = -\sum_{i=1}^{n} p_{a_i} \cdot \log_2(p_{a_i})$$



http://www.blocrawler.com/w/images/5/56/Binary_logarithm_plot.png

Symbol	Absolute Häufigkeit	p(Symbol)
a_1	3 von 6	0.5
a_2	3 von 6	0.5

e.g. E({a_1,a_1,a_1,a_2,a_2,a_2})=1 Bit

$$E(\{a_1, a_1, a_1, a_2, a_2, a_2\}) = -p_{a_1} \cdot \log_2(p_{a_1}) - p_{a_2} \cdot \log_2(p_{a_2})$$

$$= -0.5 \cdot \log_2(0.5) - 0.5 \cdot \log_2(0.5)$$

$$= 1\ Bit$$

Practise calculate number of bits for a set of messages

- Use the Matlab-method getEntropyForVector(aVector) contained in EntropyID3.zip
- Calculate the number of required bits for the vectors
  - [1;1;1;1;1;1;1;1;1]
  - [1;1;1;1]
  - [1;1;1;2;2;2]
  - [1;2;3;4;5;6]